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CHAPTER- 6
ELECTROMAGNETIC INDUCTION

The phenomena of producing induced EMF/ induced current in a circuit by changing strength, position or orientation of magnetic field is called electromagnetic induction.

The EMF so produced is called induced EMF and if the circuit has resistance, the current so produced is called induced current.

★ MAGNETIC FLUX [ϕ_B]

The number of magnetic field lines passing normally through a given surface is called magnetic flux.

$$\boxed{\phi_B = \vec{B}.\vec{A} = BA\cos\theta}$$

- Unit ⇒ Tesla-m^2 (T-m^2)
  Weber (wb) [SI Unit]
  Maxwell [CGS Unit] {1 wb = 10^8 maxwell}
- Dimensional formula ⇒ $[ML^2T^{-2}A^{-1}]$
- Nature ⇒ Scalar
- Other unit ⇒ $\frac{W}{I} = \frac{W}{q/t} = \frac{W}{q}\times t$ = Volt-sec (V-s)

• SPECIAL CASES

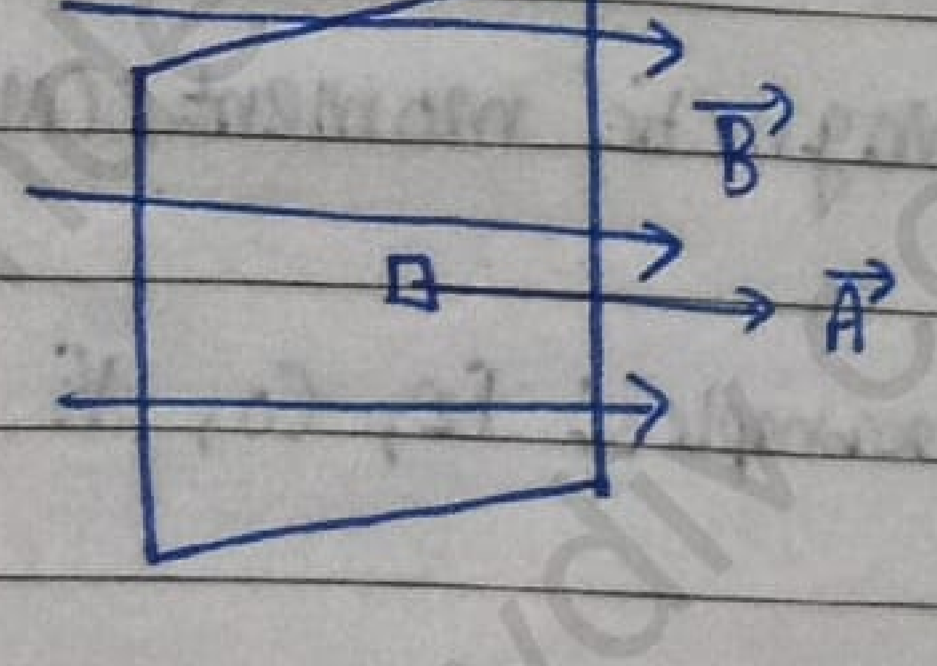
* CASE 1

When magnetic field is perpendicular to the surface

$\vec{B} \parallel \vec{A}$

$\theta = 0°$

$\boxed{\phi = BA}$



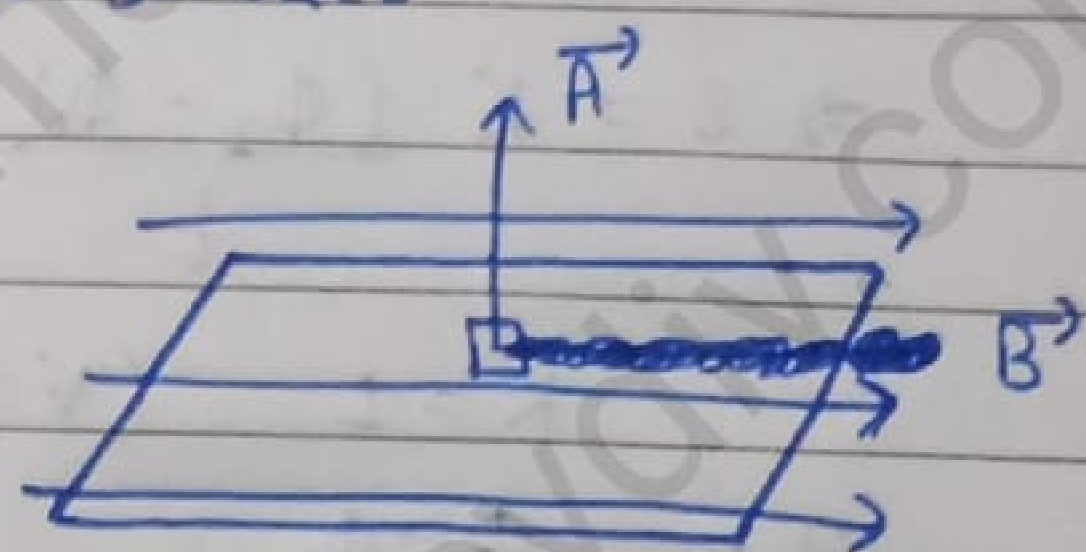
* CASE 2

When magnetic Field is parallel to the surface

$\vec{B} \perp \vec{A}$

$\theta = 90°$

$\boxed{\phi = 0}$



* CASE 3

When $\vec{B}$ and $\vec{A}$ are anti parallel to each other

$\theta = 180°$

$\boxed{\phi = -BA}$

☆ FARADAY'S LAW

- FIRST LAW

Whenever the magnetic flux linked with a circuit changes, an EMF is induced which lasts ~~to~~ as ~~so~~ long as magnetic flux changes continuously.

- SECOND LAW

The time rate of change of magnetic flux is directly proportional to the EMF induced.

☆ LENZ'S LAW

According to Lenz's law, the direction of induced current is such that it opposes the change in magnetic flux responsible for its production. (opposes its cause)

If ϕ_1 = Magnetic flux at any time

ϕ_2 = Magnetic flux after time t

According to Faraday's law,

$$e \propto \frac{\phi_2 - \phi_1}{t}$$

$$\Rightarrow e = K \frac{(\phi_2 - \phi_1)}{t}$$

$K = 1$ ($\forall$ system of units)

$$\Rightarrow e = \frac{\phi_2 - \phi_1}{t}$$

According to Lenz's law,

$$e = \frac{-(\phi_2 - \phi_1)}{t}$$

If $d\phi$ = small change in magnetic flux for a very small time dt
Then,

$$\boxed{e = -\frac{d\phi}{dt}}$$ [Faraday's law]

NOTE → If R = resistance of the circuit

$$\boxed{i = \left|\frac{e}{R}\right| = \frac{-1}{R} \times \frac{d\phi}{dt}}$$

If N = total no. of turns

$$\boxed{e = -\frac{N d\phi}{dt}}$$

- CONSEQUENCE OF LENZ'S LAW AND ENERGY CONSERVATION

Lenz's law is in accordance with the law of conservation of energy

For example, when N-pole of a magnet is moved towards a coil, the upper face of the coil acquires north polarity. Therefore, work has to be done against the force of repulsion, in bringing the magnet closer to the coil.

Similarly, when N-pole of magnet is moved away, south polarity develops on the upper ~~sur~~ face of the coil. There fore, ~~the~~ work has to be done against the force of attraction, in taking the magnet away from the coil.

This mechanical work in moving the magnet with respect to the coil changes into electrical energy producing induced current. Hence, Lenz's law obeys the principle of energy conservation.

☆ SELF - INDUCTION

The phenomena of production of induced EMF in a coil when a changing current passes through it, is called self - induction.

When a current flows through a coil, it gives rise to a magnetic flux through the coil. As the electric current strength changes, the linked magnetic flux changes and an opposing EMF induced in the coil. This EMF is called self-induced ~~current~~. EMF or back EMF, and this phenomena is called self-induction.

• COEFFICIENT OF SELF - INDUCTION [SELF - INDUCTANCE]

The magnetic flux linked with a coil is directly proportional to the current through it.

$$\phi \propto i$$

$$\boxed{\phi = Li}$$

OR

$$\boxed{L = \frac{\phi}{i}}$$

where, l = coefficient of self-induction (self inductance)

Joule / A^2 = $\frac{W}{(q/t)^2} = \frac{W \times t^2}{q}$

- Unit ⇒ Weber / ampere ($wb\ A^{-1}$)
- SI Unit ⇒ Henry (H)
- Dimensional formula ⇒ $[ML^2T^{-2}A^{-2}]$

∴ other unit ⇒ Volt - sec^2

NOTE → According to Faraday's law,

$$e = -\frac{d\phi}{dt} \quad \text{—— ①}$$

and we have,

$$\phi = Li \quad \text{—— ②}$$

From ① and ②,

$$e = -\frac{d(Li)}{dt}$$

$$\Rightarrow \boxed{e = -L\frac{di}{dt}}$$

Definition of one henry

$$|e| = L\frac{di}{dt}$$

$$\Rightarrow L = \frac{e}{\frac{di}{dt}}$$

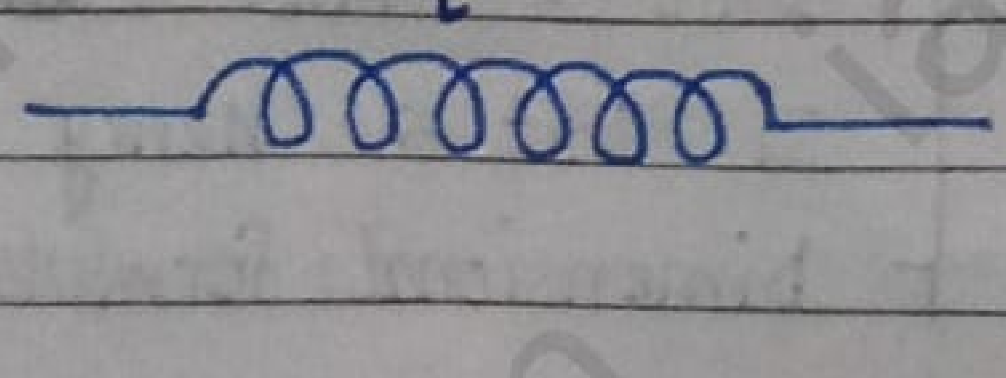
If $e = 1$ volt and $\frac{di}{dt} = 1$ A/s

then $L = \frac{1 \text{ volt}}{1 \text{ A/s}}$

$= 1$ Henry

∴ The self-inductance of a coil is said to be one henry when one volt EMF is induced in the coil, if the electric current changes by a rate of 1 A/s.

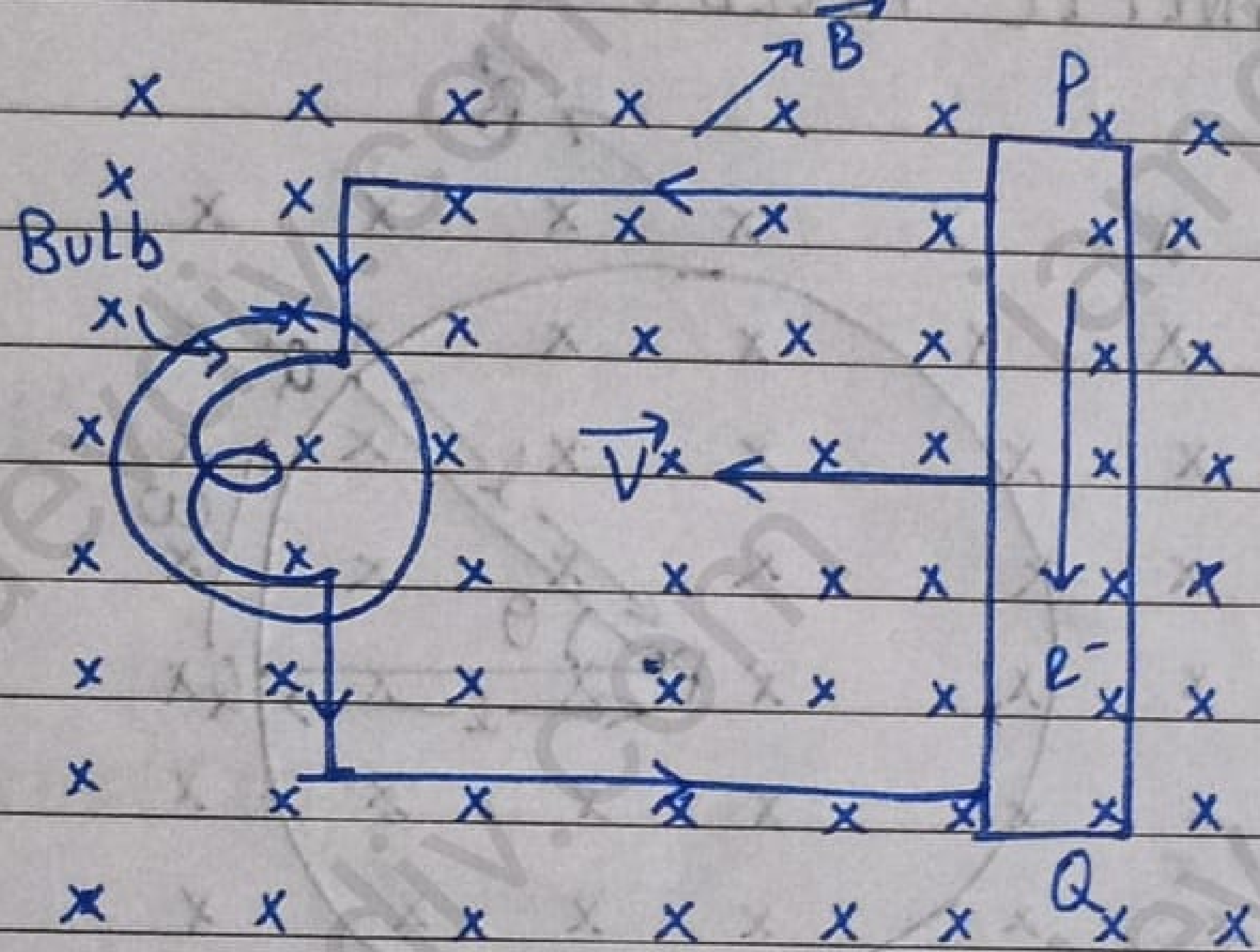
Symbol of induction



~~MUTUAL INDUCTION~~

★ MOTIONAL EMF

When a conducting rod is moved in an uniform magnetic field, an EMF is induced in it, this EMF is called motional EMF.



Let's consider a conductor PQ of length l is moving with velocity $\vec{v}$ in uniform magnetic field $\vec{B}$

The work done in moving the charge from P to Q,

$$W = \text{Force} \times \text{displacement}$$

$$\Rightarrow W = qVB\sin\theta \times PQ$$

$$\because \theta = 90^\circ$$

$$\vec{v} \perp \vec{B}$$

$$PQ = l$$

$$\Rightarrow W = qVBl \quad \text{--- (1)}$$

We know that,

$$EMF = \frac{\text{Work done}}{\text{Charge}}$$

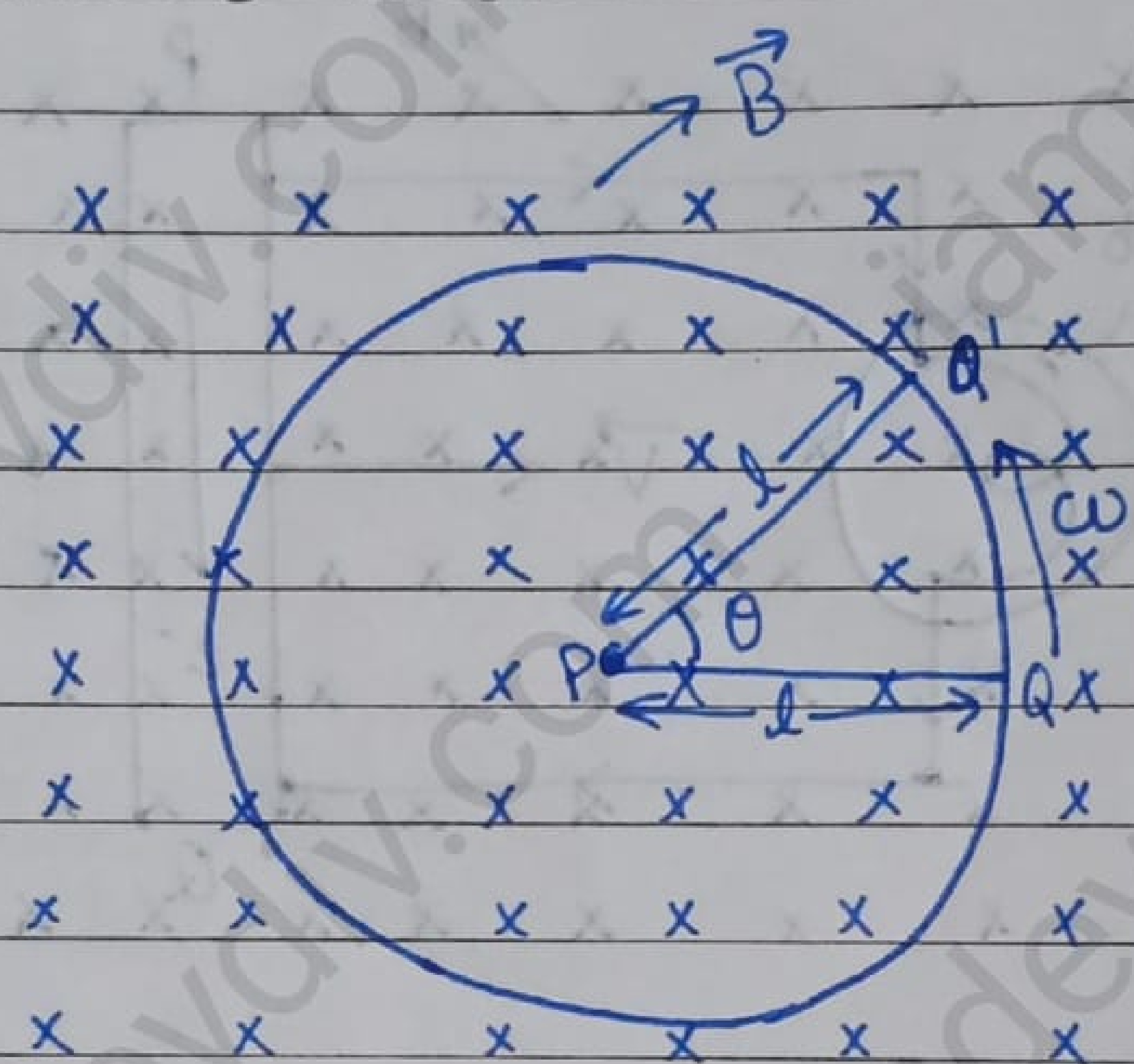
$$\Rightarrow e = \frac{W}{q} = \frac{\cancel{q}VBl}{\cancel{q}}$$

$$\Rightarrow \boxed{e = Blv}$$

if R = resistance of conductor,

$$\boxed{i = \frac{e}{R} = \frac{Blv}{R}}$$

- MOTIONAL INDUCED EMF IN A CONDUCTOR BY ROTATION IN UNIFORM MAGNETIC FIELD



Let's consider a conductor PQ of length l is rotating with angular velocity ω in an uniform magnetic field $\vec{B}$

Area swept by the conductor,

$$A = \frac{1}{2} \times PQ \times PQ'$$

$$\Rightarrow A = \frac{1}{2} l \times QQ' \quad \text{--- (1)}$$

In $\Delta Q'PQ$,

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}}$$

$$\theta = \frac{QQ'}{l}$$

$$\Rightarrow QQ' = \theta l$$

$$① \Rightarrow A = \frac{1}{2} l^2 \theta \quad \text{——} ②$$

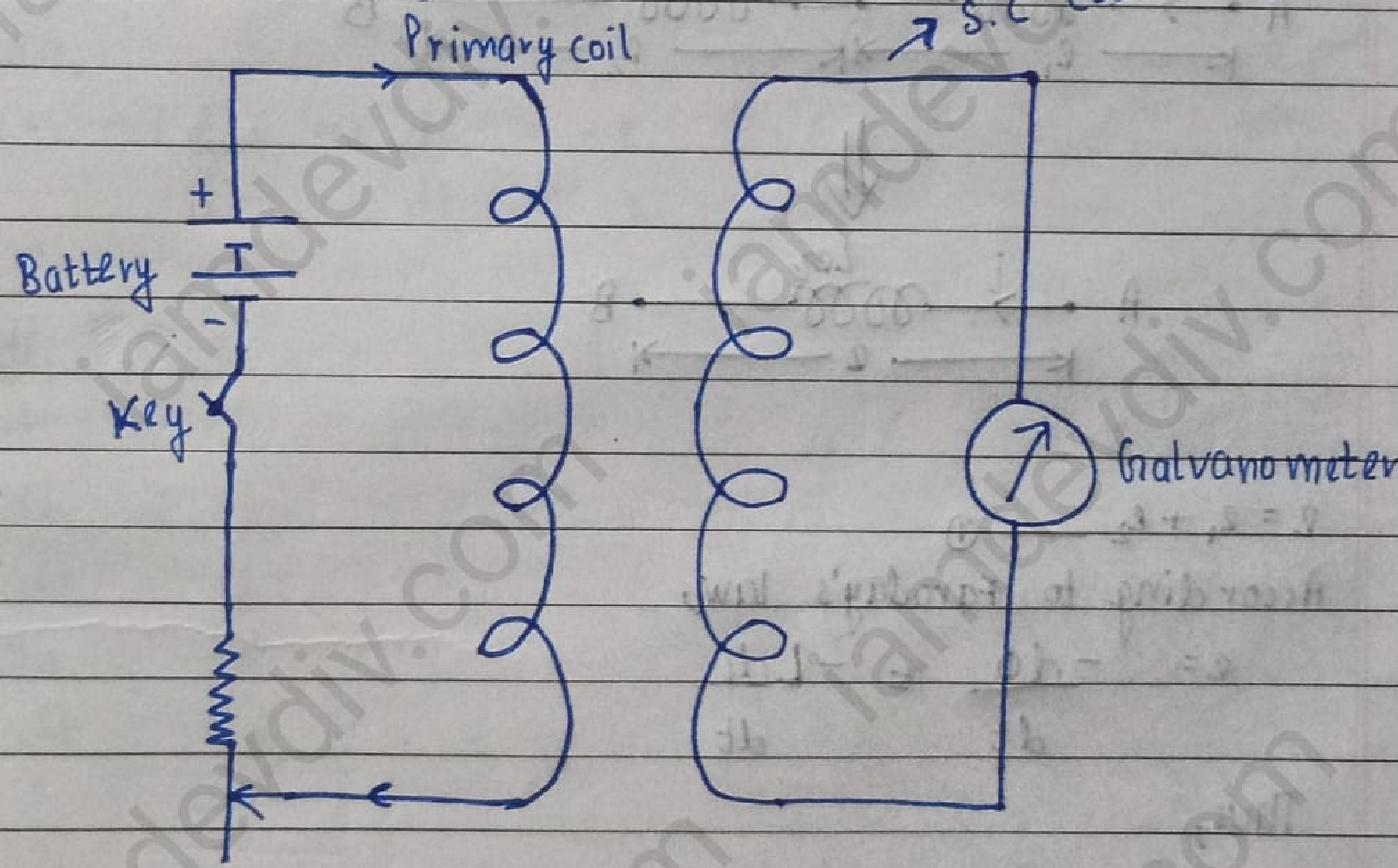
Magnetic flux, $\Phi = BA$

$$\Phi = \frac{1}{2} l^2 B \theta$$

$$e = \frac{d\Phi}{dt} = \frac{1}{2} l^2 B \frac{d\theta}{dt}$$

$$e = \frac{1}{2} B l^2 \omega$$

☆ MUTUAL INDUCTION.



The phenomena of production of induced EMF in one coil due to the change in current in its neighbouring coil is called mutual induction.

here,

magnetic flux linked with secondary coil is directly proportional to current flowing in the primary coil

$\phi \propto i$

$\Rightarrow \phi = Mi$

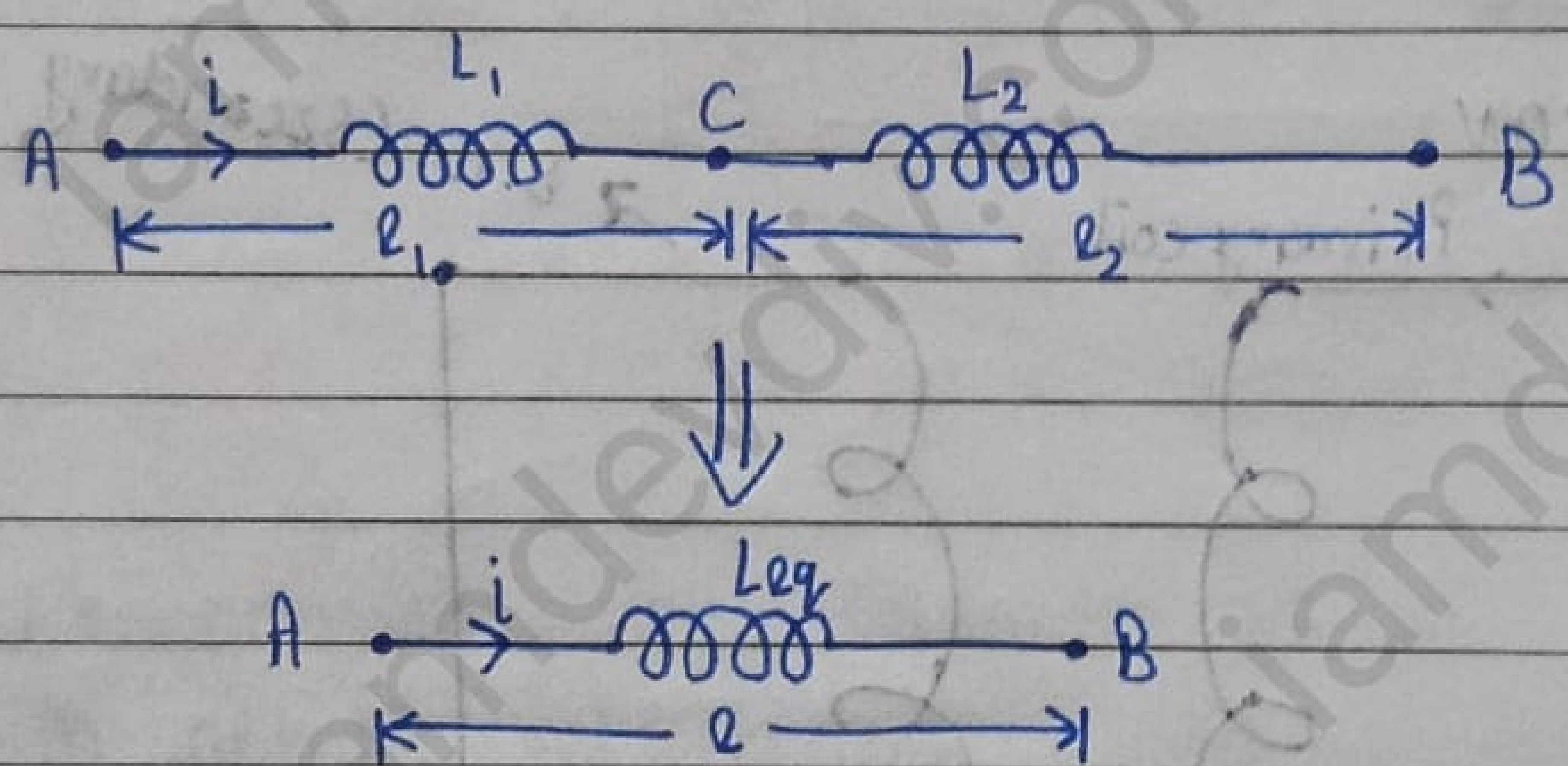
$\Rightarrow M = \dfrac{\phi}{i}$

→ Coefficient of mutual induction (Mutual inductance)

→ Unit ⇒ henry (H)

★ GROUPING OF COILS

- IN SERIES



$e = e_1 + e_2$ —— (1)

According to Faraday's law,

$e = -\dfrac{d\phi}{dt} = -L\dfrac{di}{dt}$

Thus,

$e_1 = -L_1 \dfrac{di}{dt}$

$e_2 = -L_2 \dfrac{di}{dt}$

$e_{eq} = -L_{eq} \dfrac{di}{dt}$

$-L_{eq}\dfrac{di}{dt} = -L_1\dfrac{di}{dt} - L_2\dfrac{di}{dt}$

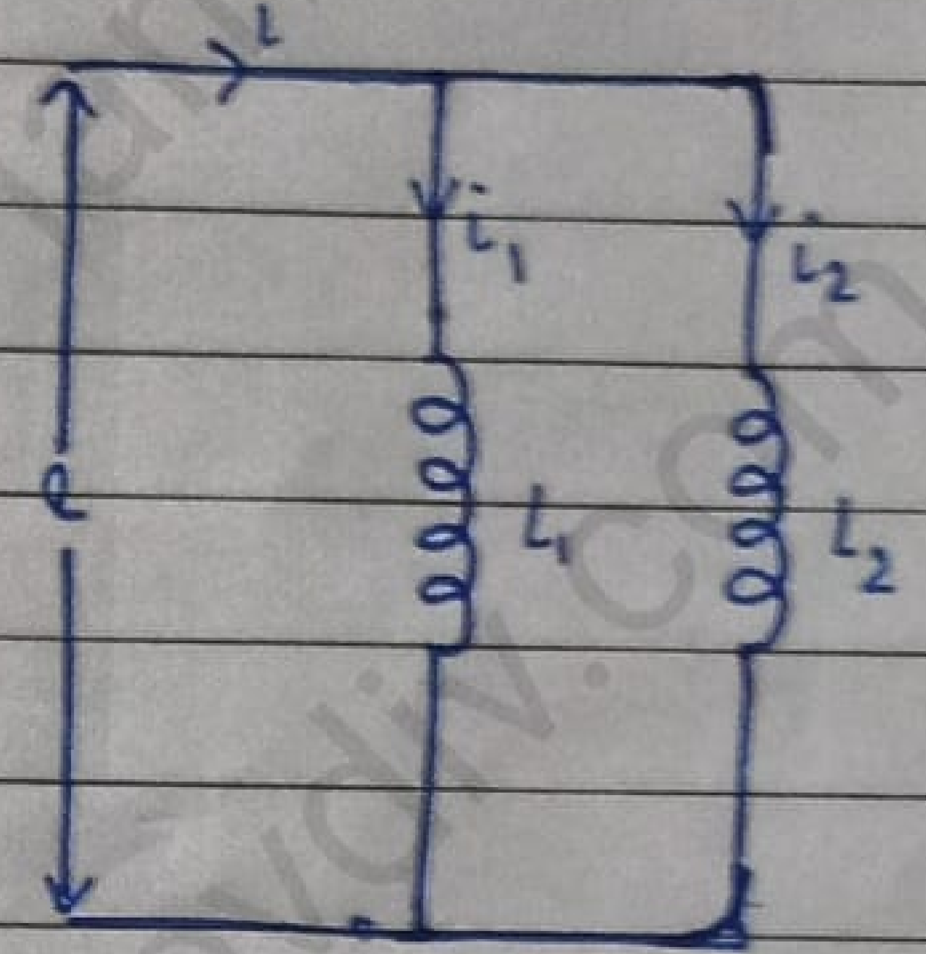
$$\boxed{L_{eq} = L_1 + L_2}$$

NOTE → If N coils of self inductances $L_1, L_2, \ldots L_n$ are connected in series

$$L_{eq} = L_1 + L_2 + \ldots + L_n$$

- IN PARALLEL

$$i = i_1 + i_2 \quad \text{--- (1)}$$



Differentiating w.r.t 't',

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \text{--- (2)}$$

We have,

$$e = -L\frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{-e}{L}$$

Thus,

$$\frac{di_1}{dt} = \frac{-e}{L_1}$$

$$\frac{di_2}{dt} = \frac{-e}{L_2}$$

$$\frac{di}{dt} = \frac{-e}{L_{eq}}$$

$$\frac{-e}{L_{eq}} = \frac{-e}{L_1} - \frac{e}{L_2}$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

NOTE → If N coils $L_1, L_2, L_3, \ldots L_n$ are connected in parallel,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{L_n}$$

★ SELF- INDUCTANCE OF A LONG SOLENOID

$B = \mu_0 n i$

$\because n = \frac{N}{l}$

$\Rightarrow B = \frac{\mu_0 N i}{l}$

$\phi = NBA = \frac{\mu_0 N^2 i}{l} A$ ——— ①

Also,

$\phi = Li$ ——— ②

From eq^n ① and ②

$L\cancel{i} = \frac{\mu_0 N^2 \cancel{i} A}{l}$

$\Rightarrow \boxed{L = \frac{\mu_0 N^2 A}{l}}$

★ AC GENERATOR / AC DYNAMO / ALTERNATOR

It is a device used to convert mechanical energy into electrical energy and is based on the phenomenon on electromagnetic induction.

- EMF GENERATOR

We have,

$\phi = BA \cos\theta$ ——— ①

According to Faraday's law,

$e = -\frac{d\phi}{dt}$ ——— ②

From ① and ②

$$e = -\frac{d}{dt}(BA\cos\theta)$$

$$\because \omega = \frac{\theta}{t} \Rightarrow \theta = \omega t$$

$$\Rightarrow e = -\frac{d}{dt}(BA\cos\omega t)$$

$$\Rightarrow e = \omega BA \sin\omega t$$

If N = total number of turns

$$\Rightarrow e = N\omega BA \sin\omega t$$

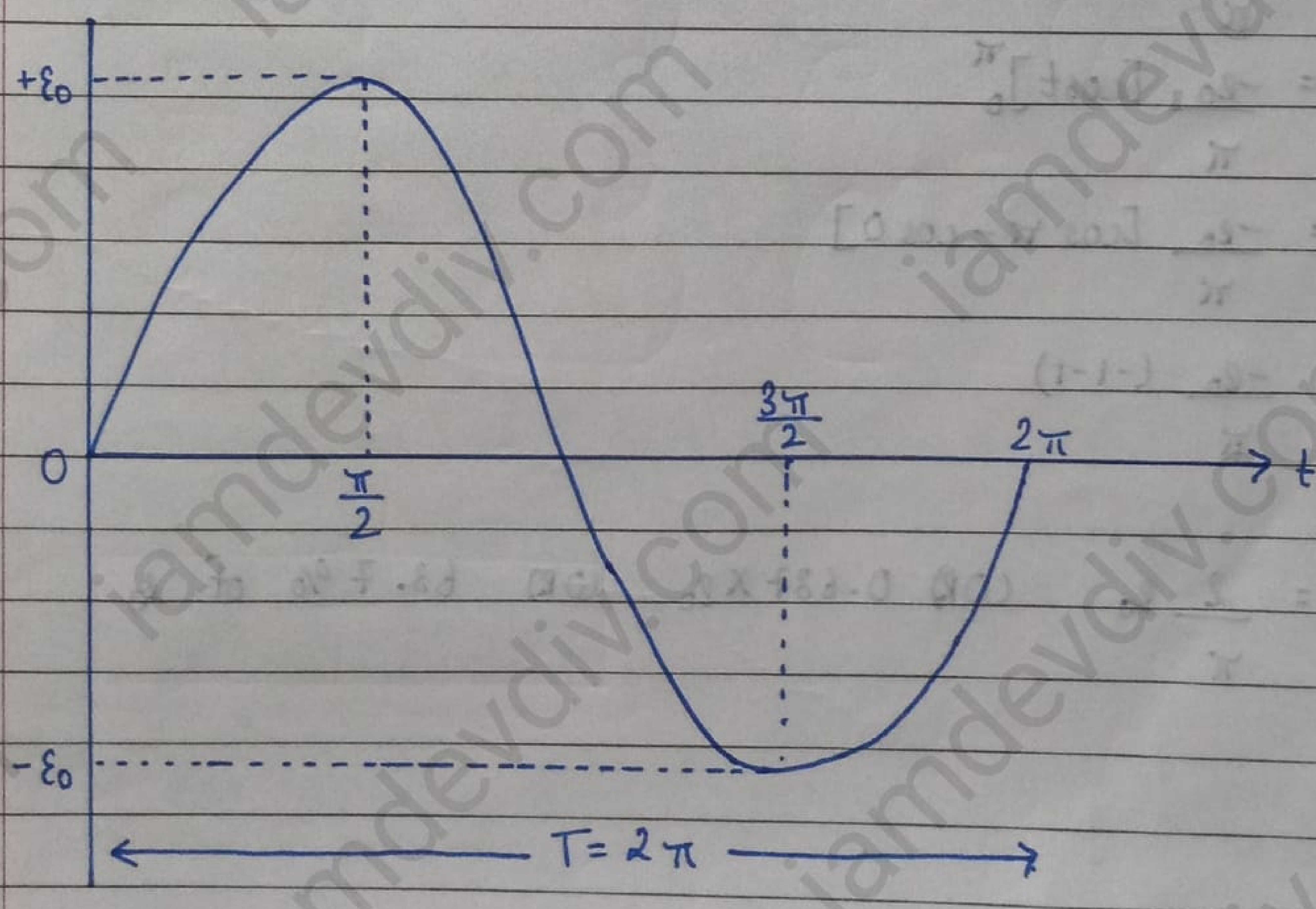
$$\because N\omega BA = e_0$$

$$\Rightarrow \boxed{e = e_0 \sin\omega t}$$

NOTE → If R = resistance of coil

$$\frac{e}{R} = \frac{e_0}{R}\sin\omega t$$

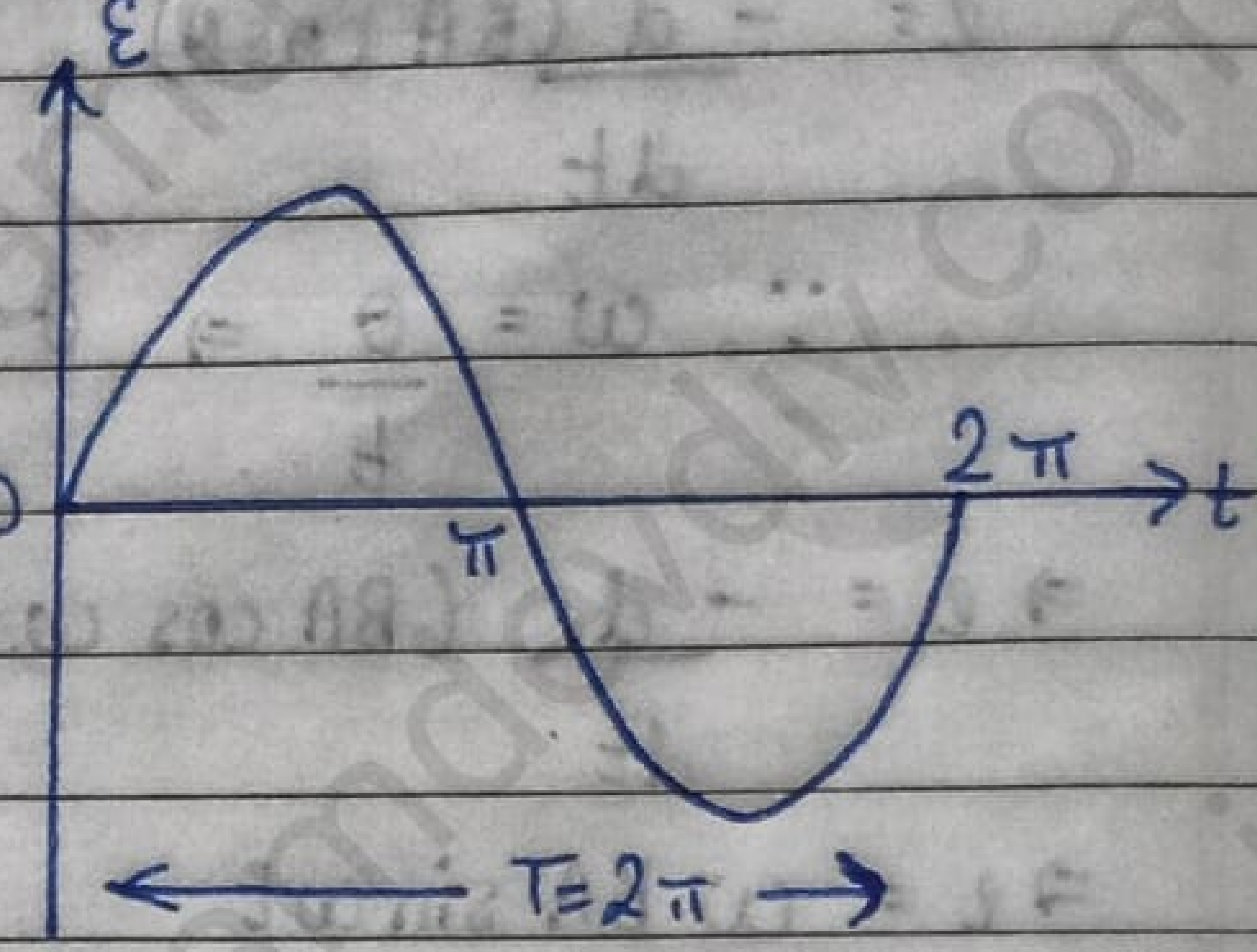
$$\Rightarrow \boxed{i = i_0 \sin\omega t}$$



☆ AVERAGE (MEAN) OF AN AC

$e = e_0 \sin \omega t$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1 \text{ rad/s}$



$\Rightarrow e = e_0 \sin t$

We know that,

the average value of an AC over complete cycle is zero

So,

average value of an AC over half cycle is

$e_{avg} = \frac{1}{T} \int_0^t e\,dt$

$= \frac{1}{\pi} \int_0^\pi e_0 . \sin t\,dt$

$= \frac{e_0}{\pi} \int_0^\pi \sin t . dt$

$= \frac{-e_0}{\pi} [\cos t]_0^\pi$

$= \frac{-e_0}{\pi} [\cos \pi - \cos 0]$

$= \frac{-e_0}{\pi} (-1-1)$

$\Rightarrow e_{avg} = \frac{2}{\pi} e_0$ (OR) $0.637 \times e_0$ (OR) 63.7% of e_0

☆ ~~LARANTZ~~ LORENTZ FORCE

It is the force experienced by a charge in electric field and magnetic field

$F_e = qE$

$F_m = qVB\sin\theta$

$F = qE + qVB\sin\theta$

$= q(E + VB\sin\theta)$

* CASE - 1

When charge is perpendicular to magnetic field

$\theta = 90°$

$F = q(E + VB\sin 90°)$

$= q(E + VB)$

* CASE - 2

When charge is parallel to magnetic field

$\theta = 0°$

$F = q(E + VB\sin 0°)$

$= qE$

* CASE - 3

When charge is anti-parallel to magnetic field

$\theta = 180°$

$F = q(E + VB\sin 180°)$

$= qE$